SM3 12.3: Pyth Trig Proof

1)

$2\cos^2\theta - 1 = 1 - 2\sin^2\theta$	Given
$2(1-\sin^2\theta) - 1 =$	Pyth ID
$2 - 2\sin^2\theta - 1 =$	Distrib
$1 - 2\sin^2\theta =$	Add

QED

2)

$\tan^2\theta = \dfrac{1-\cos^2\theta}{\cos^2\theta}$	Given
$\dfrac{\sin^2\theta}{\cos^2\theta} =$	Def of tan
$\dfrac{1-\cos^2\theta}{\cos^2\theta} =$	Pyth ID

QED

3)

$4\sin^2\theta + 4\cos^2\theta = 4$	Given
$4(\sin^2\theta + \cos^2\theta) =$	Factor
$4 =$	Pyth ID

QED

4)

$\cos\theta - \cos^3\theta = \cos\theta\sin^2\theta$	Given
$\cos\theta(1-\cos^2\theta) =$	Factor
$\cos\theta\sin^2\theta =$	Pyth ID

QED

5)

$\dfrac{\cos^2\theta - 1}{\cos\theta} = -\tan\theta\sin\theta$	Given
$\dfrac{-(1-\cos^2\theta)}{\cos\theta} =$	Factor
$\dfrac{-\sin^2\theta}{\cos\theta} =$	Pyth ID
$\dfrac{-\sin\theta\sin\theta}{\cos\theta} =$	Factor
$-\tan\theta\sin\theta =$	Def of tan

QED

6)

$\dfrac{\sec\theta + 1}{\tan\theta} = \dfrac{\sin\theta}{1-\cos\theta}$	Given
$\dfrac{\frac{1}{\cos\theta} + 1}{\frac{\sin\theta}{\cos\theta}} =$	Def of sec,tan
$\left(\dfrac{1}{\cos\theta} + 1\right)\left(\dfrac{\cos\theta}{\sin\theta}\right) =$	Division
$\left(\dfrac{1+\cos\theta}{\cos\theta}\right)\left(\dfrac{\cos\theta}{\sin\theta}\right) =$	Add
$\dfrac{1+\cos\theta}{\sin\theta} =$	Mult
$\dfrac{(1+\cos\theta)\sin\theta}{\sin^2\theta} =$	Mult
$\dfrac{(1+\cos\theta)\sin\theta}{1-\cos^2\theta} =$	Pyth ID
$\dfrac{(1+\cos\theta)\sin\theta}{(1-\cos\theta)(1+\cos\theta)} =$	Factor
$\dfrac{\sin\theta}{1-\cos\theta} =$	Divide

QED

7) $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$ Given

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) =$ Factor

$\cos^2\theta - \sin^2\theta =$ Pyth ID

QED

8) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$ Given

$\tan^2\theta(\tan^2\theta + 1) =$ Factor

$(\sec^2\theta - 1)(\sec^2\theta) =$ Pyth ID

$\sec^4\theta - \sec^2\theta =$ Distrib

QED

9) $(1 - \tan\theta)^2 = \sec^2\theta - 2\tan\theta$ Given

$1 - 2\tan\theta + \tan^2\theta =$ Distribute

$\sec^2\theta - 2\tan\theta =$ Pyth ID

QED

10) $(\cos\theta - \sin\theta)^2 = 1 - 2\sin\theta\cos\theta$ Given

$\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta =$ Distrib

$1 - 2\sin\theta\cos\theta =$ Pyth ID

QED

11) $\dfrac{\cos^2\theta}{1 - \sin\theta} = 1 + \sin\theta$ Given

$\dfrac{1 - \sin^2\theta}{1 - \sin\theta} =$ Pyth ID

$\dfrac{(1 - \sin\theta)(1 + \sin\theta)}{1 - \sin\theta} =$ Factor

$1 + \sin(\theta) =$ Divide

QED

12) $(\sec^2\theta + \csc^2\theta) - (\tan^2\theta + \cot^2\theta) = 2$ Given

$\sec^2\theta - \tan^2\theta + \csc^2\theta - \cot^2\theta =$ Distrib

$1 + 1 =$ Pyth ID

$2 =$ Add

QED

13) $$\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}=\frac{2}{\sin^2\theta}$$ Given

$$\frac{1+\cos\theta}{1-\cos^2\theta}+\frac{1-\cos\theta}{1-\cos^2\theta}=$$ Mult

$$\frac{2}{1-\cos^2\theta}=$$ Add

$$\frac{2}{\sin^2\theta}=$$ Pyth ID

QED

14) $$\frac{\sec^2\theta\csc\theta}{\sec^2\theta+\csc^2\theta}=\sin\theta$$ Given

$$\frac{\frac{1}{\cos^2\theta\sin\theta}}{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}=$$ Def of sec, csc

$$\frac{\frac{1}{\cos^2\theta\sin\theta}}{\frac{\sin^2\theta}{\cos^2\theta\sin^2\theta}+\frac{\cos^2\theta}{\cos^2\theta\sin^2\theta}}=$$ Mult

$$\frac{\frac{1}{\cos^2\theta\sin\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sin^2\theta}}=$$ Add

$$\frac{\frac{1}{\cos^2\theta\sin\theta}}{\frac{1}{\cos^2\theta\sin^2\theta}}=$$ Pyth ID

$$\frac{1}{\cos^2\theta\sin\theta}\frac{\cos^2\theta\sin^2\theta}{1}=$$ Divide

$$\sin\theta=$$ Divide

QED

15) $$\tan^4\theta=\tan^2\theta\sec^2\theta-\sec^2\theta+1$$ Given

$$\tan^2\theta\tan^2\theta=$$ Factor

$$\tan^2\theta\,(\sec^2\theta-1)=$$ Pyth ID

$$\tan^2\theta\sec^2\theta-\tan^2\theta=$$ Distribute

$$\tan^2\theta\sec^2\theta-(\sec^2\theta-1)=$$ Pyth ID

$$\tan^2\theta\sec^2\theta-\sec^2\theta+1=$$ Distribute

QED